Some Bianchi Type Cosmological Models in $f(R)$ Gravity

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Abstract

The modified theories of gravity, especially the $f(R)$ gravity, have attracted much attention in the last decade. In this context, we study the exact vacuum solutions of Bianchi type I, III and Kantowski-Sachs spacetimes in the metric version of $f(R)$ gravity. The field equations are solved by taking expansion scalar θ proportional to shear scalar σ which gives $A = B^n$, where A and B are the metric coefficients. The physical behavior of the solutions has been discussed using some physical quantities. Also, the function of the Ricci scalar is evaluated in each case.

1 Introduction

The late time accelerated expansion of the universe has attracted much attention in the recent years. Direct evidence of cosmic acceleration comes from high red-shift supernova experiments [1]. Some other observations, such as cosmic microwave background fluctuations [2] and large scale structure [3],

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provide an indirect evidence. These observations seem to change the entire picture of our matter filled universe. It is now believed that most part of the universe contains dark matter and dark energy. The modifications of general relativity (GR) seem attractive to explain late time acceleration and dark energy.

Among the various modifications of GR, the $f(R)$ theory of gravity is treated most seriously during the last decade. It provides a natural gravitational alternative to dark energy. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein-Hilbert action of GR with a general function Ricci scalar $f(R)$. $f(R)$ theory of gravity has been shown [4] equivalent to scalar-tensor theory of gravity that is incompatible with solar system tests of GR, as long as the scalar field propagates over solar system scales. The explanation of cosmic acceleration is obtained just by introducing the term $1/R$ which is essential at small curvatures. Capozziello et al. [5] have shown that dust matter and dark energy phases can be achieved by the exact solution derived from a power law $f(R)$ cosmological model. The $f(R)$ theory of gravity is considered most suitable due to cosmologically important $f(R)$ models. These models consist of higher order curvature invariants as functions of the Ricci scalar. Viable $f(R)$ gravity models [6] have been proposed which show the unification of early-time inflation and late-time acceleration. The problem of dark matter can also be addressed by using viable $f(R)$ gravity models. There are some other useful aspects [7] of $f(R)$ gravity. It gives an easy unification of early time inflation and late time acceleration. It can be used for the explanation of hierarchy problem in high energy physics. It also describes the transition phase of the universe from deceleration to acceleration. Thus $f(R)$ theory of gravity seems attractive and a reasonable amount of work has been done in different contexts.

Lobo and Oliveira [8] constructed wormhole geometries in the context of $f(R)$ theories of gravity. Cognola et al. [9] investigated $f(R)$ gravity at one-loop level in de-Sitter universe. It was found that one-loop effective action can be useful for the study of constant curvature black hole nucleation rate. Multamäki and Vilja [10] investigated spherically symmetric vacuum solutions in $f(R)$ theory. The same authors [11] also studied the perfect fluid solutions and showed that pressure and density did not uniquely determine $f(R)$. In a recent paper, Sharif and Kausar [12] studied non-vacuum static spherically symmetric solutions in this theory. Capozziello et al. [13] explored spherically symmetric solutions of $f(R)$ theories of gravity via the Noether symmetry approach. Hollenstein and Lobo [14] analyzed exact so-

lutions of static spherically symmetric spacetimes in $f(R)$ gravity coupled to non-linear electrodynamics. Azadi et al. [15] studied cylindrically symmetric vacuum solutions in this theory. Momeni and Gholizade [16] extended cylindrically symmetric solutions in a more general way. Reboucas and Santos [17] studied Gödel-type universes in f(R) gravity. We have explored static plane symmetric vacuum solutions [18] in $f(R)$ gravity. The field equations are solved using the assumption of constant scalar curvature which may be zero or non-zero. In a recent paper, Babichev and Langlois [19] studied relativistic stars in this theory.

Friedmann-Robertson-Walker (FRW) models, being spatially homogeneous and isotropic in nature, are best for the representation of the large scale structure of the present universe. However, it is believed that the early universe may not have been exactly uniform. Thus, the models with anisotropic background are the most suitable to describe the early stages of the universe. Bianchi type models are among the simplest models with anisotropic background. Many authors [20]-[27] explored Bianchi type spacetimes in different contexts. Kumar and Singh [28] studied solutions of the field equations in the presence of perfect fluid using Bianchi type I spacetime in GR. Moussiaux et al. [29] investigated the exact solution for vacuum Bianchi type-III model with a cosmological constant. Lorenz-Petzold [20] studied exact Bianchi type-III solutions in the presence of electromagnetic field. Xing-Xiang [30] discussed Bianchi type III string cosmology with bulk viscosity. He assumed that the expansion scalar is proportional to the shear scalar to derive the solutions. Wang [31] investigated string cosmological models with bulk viscosity in Kantowski-Sachs spacetime. Upadhaya [32] explored some magnetized Bianchi type-III massive string cosmological models in GR. Recently, Hellaby [33] presented an overview of some recent developments in inhomogeneous models and it was concluded that the universe is inhomogeneous on many scales.

The investigation of Bianchi type models in alternative or modified theories of gravity is also an interesting discussion. Kumar and Singh [34] investigated perfect fluid solutions using Bianchi type I spacetime in scalar-tensor theory. Singh et al. [35] studied some Bianchi type-III cosmological models in scalar-tensor theory. Adhav et al. [36] obtained an exact solution of the vacuum Brans-Dicke field equations for the metric tensor of a spatially homogeneous and anisotropic model. Paul et al. [37] investigated FRW cosmologies in $f(R)$ gravity. Recently, we [38, 39] have studied the solutions of Bianchi types I and V spacetimes in the framework of $f(R)$ gravity.

In this paper, we focuss our attention to explore the vacuum solutions of Bianchi types I, III and Kantowski-Sachs spacetimes in metric $f(R)$ gravity. The field equations are solved by taking expansion scalar θ proportional to shear scalar σ which gives $A = B^n$, where A and B are the metric coefficients. The paper is organized as follows: A brief introduction of the field equations in metric version of $f(R)$ gravity is given in section **2**. In section **3**, **4** and **5**, the solutions of the field equations for Bianchi types I, III and Kantowski-Sachs spacetimes are found. Some physical parameters and the functions of Ricci scalar are also evaluated in the context of these solutions. In the last section, we discuss the results.

2 Some Basics of $f(R)$ Gravity

The metric tensor plays an important role in GR. The dependence of Levi-Civita connection on the metric tensor is one of the main properties of GR. However, if we allow torsion in the theory, then the connection no longer remains the Levi-Civita connection and the dependence of connection on the metric tensor vanishes. This is the main idea behind different approaches of $f(R)$ theories of gravity.

When the connection is the Levi-Civita connection, we get metric $f(R)$ gravity. In this approach, we take variation of the action with respect to the metric tensor only. The action for $f(R)$ gravity is given by

$$S = \int \sqrt{-g}(f(R) + L_m)d^4x, \tag{1}$$

where $f(R)$ is a general function of the Ricci scalar and L_m is the matter Lagrangian. The field equations resulting from this action are

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = T^m_{\mu\nu}, \tag{2}$$

where $F(R) \equiv df(R)/dR$, $\Box \equiv \nabla^\mu\nabla_\mu$, ∇_μ is the covariant derivative and $T^m_{\mu\nu}$ is the standard minimally coupled stress energy tensor derived from the Lagrangian L_m. Now contracting the field equations, it follows that

$$F(R)R - 2f(R) + 3\Box F(R) = T$$

In vacuum, this reduces to

$$F(R)R - 2f(R) + 3\Box F(R) = 0. \tag{3}$$

which implies that
$$f(R) = \frac{3\Box F(R) + F(R)R}{2}. \tag{4}$$
This gives an important relationship between $f(R)$ and $F(R)$ which will be used to simplify the field equations and to evaluate $f(R)$.

Now we consider the metric
$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 + l^2(\theta)d\phi^2], \tag{5}$$
where
$$l^2(\theta) = \begin{cases} \theta^2 & \text{when } k=0 \quad \text{(Bianchi I model)}, \\ \sinh^2\theta & \text{when } k=-1 \quad \text{(Bianchi III model)}, \\ \sin^2\theta & \text{when } k=1 \quad \text{(Kantowski-Sachs model)}. \end{cases}$$

Here k is the spatial curvature index and the above three models are Euclidian, semi-closed and closed respectively.

3 Bianchi Type I Solution

Here we shall find exact solutions of Bianchi I spacetime in $f(R)$ gravity. For the sake of simplicity, we take the vacuum field equations. The line element of Bianchi type I spacetime is given by
$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 + \theta^2 d\phi^2], \tag{6}$$
where A and B are cosmic scale factors. The corresponding Ricci scalar is given by
$$R = -2[\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}], \tag{7}$$
where dot denotes derivative with respect to t. Using Eq.(4), the vacuum field equations take the form,
$$\frac{F(R)R_{\mu\nu} - \nabla_\mu\nabla_\nu F(R)}{g_{\mu\nu}} = \frac{F(R)R - \Box F(R)}{4}. \tag{8}$$
One can view Eq.(8) as the set of differential equations for $F(t)$, A and B. It follows from Eq.(8) that the combination
$$A_\mu \equiv \frac{F(R)R_{\mu\mu} - \nabla_\mu\nabla_\mu F(R)}{g_{\mu\mu}}, \tag{9}$$

is independent of the index μ and hence $A_\mu - A_\nu = 0$ for all μ and ν. Thus $A_0 - A_1 = 0$ gives

$$-\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} = 0. \tag{10}$$

Also, $A_0 - A_2 = 0$ yields

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0. \tag{11}$$

Now we give definition of some physical quantities before solving these equations.

The average scale factor a and the volume scale factor V are defined as

$$a = \sqrt[3]{AB^2}, \quad V = a^3 = AB^2. \tag{12}$$

The average Hubble parameter H is given in the form

$$H = \frac{1}{3}(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}). \tag{13}$$

The expansion scalar θ and shear scalar σ are defined as follows

$$\theta = u^\mu_{;\mu} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}]^2, \tag{15}$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}h^\alpha_\nu + u_{\nu;\alpha}h^\alpha_\mu) - \frac{1}{3}\theta h_{\mu\nu}, \tag{16}$$

$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is the projection tensor while $u_\mu = \sqrt{g_{00}}(1,0,0,0)$ is the four-velocity in co-moving coordinates.

We have two differential equations given by Eqs.(10,11) with three unknowns namely A, B and F. Thus we need one additional constraint to solve these equations. We use a physical condition that expansion scalar θ is proportional to shear scalar σ which gives

$$A = B^n. \tag{17}$$

Using this condition, Eqs.(10,11) take the form

$$-\frac{2\ddot{B}}{B} + 2n\frac{\dot{B}^2}{B^2} + n\frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0, \tag{18}$$

$$(n+1)\frac{\ddot{B}}{B} + (n^2 - 2n - 1)\frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{F}}{BF} + \frac{\ddot{F}}{F} = 0. \tag{19}$$

Adding these, we get

$$\frac{\ddot{B}}{B} + (n+1)\frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} = 0. \tag{20}$$

We solve this equation using power law relation between F and a [38],

$$F = ka^m, \tag{21}$$

where k is the constant of proportionality, m is any integer and a is given by

$$a = B^{\frac{n+2}{3}} \tag{22}$$

Thus for $m = 3$, we obtain

$$F = kB^{n+2}. \tag{23}$$

Using this in Eq.(20), it follows that

$$\frac{\ddot{B}}{B} + (2n+3)\frac{\dot{B}^2}{B^2} = 0. \tag{24}$$

Put $\dot{B} = g(B)$ in this equation, we get

$$\frac{dg^2}{dB} + \frac{4n+6}{B}g^2 = 0, \tag{25}$$

which leads to the solution

$$g^2 = \frac{c_1}{B^{4n+6}}, \tag{26}$$

where c_1 is an integration constant. Hence the solution becomes

$$ds^2 = (\frac{dt}{dB})^2 dB^2 - B^{2n}dr^2 - B^2(d\theta^2 + \theta^2 d\phi^2), \tag{27}$$

which can be written as

$$ds^2 = \frac{1}{c_1}B^{4n+6}dB^2 - B^{2n}dr^2 - B^2(d\theta^2 + \theta^2 d\phi^2), \tag{28}$$

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and after the transformations $B = T$, $r = R$, $\theta = \Theta$ and $\phi = \Phi$, it takes the form

$$ds^2 = \frac{1}{c_1}T^{4n+6}dT^2 - T^{2n}dR^2 - T^2(d\Theta^2 + \Theta^2 d\Phi^2). \tag{29}$$

The average Hubble parameter becomes

$$H = \frac{\sqrt{c_1}(n+2)}{3T^{2n+4}}. \tag{30}$$

while the volume scale factor turns out to be

$$V = T^{n+2}. \tag{31}$$

The expansion scalar θ is given by

$$\theta = \frac{\sqrt{c_1}(n+2)}{T^{2n+4}}, \tag{32}$$

while the shear scalar σ becomes

$$\sigma^2 = \frac{c_1(n-1)^2}{3T^{4n+8}}. \tag{33}$$

Moreover, the function of Ricci scalar, $f(R)$, can be found by using Eq.(4)

$$f(R) = \frac{kT^{n+2}R}{2}. \tag{34}$$

It follows from Eq.(7) that

$$R = 2c_1(n^2 + 6n + 5)T^{-4n-8}. \tag{35}$$

Thus $f(R)$ can be written as a function of R only

$$f(R) = \frac{k}{2}[2c_1(n^2 + 6n + 5)]^{\frac{n+2}{4n+8}}R^{\frac{3}{4}}. \tag{36}$$

4 Bianchi Type III Solution

Here we shall find the solution of the Bianchi type III spacetime in $f(R)$ gravity for the vacuum field equations. The line element of Bianchi type III spacetime is given by

$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 + \sinh^2\theta d\phi^2], \tag{37}$$

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where A and B are cosmic scale factors. The Ricci scalar for this spacetime is given by

$$R = -2[\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} - \frac{1}{B^2} + \frac{\dot{B}^2}{B^2}], \tag{38}$$

where dot denotes derivative with respect to t. Using Eq.(9), the vacuum field equations take the form,

$$-\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} = 0, \tag{39}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0. \tag{40}$$

Here we also need one additional constraint to solve these equations. Using the same physical condition that expansion scalar θ is proportional to shear scalar σ, we obtain

$$-\frac{2\ddot{B}}{B} + 2n\frac{\dot{B}^2}{B^2} + n\frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0, \tag{41}$$

$$(n+1)\frac{\ddot{B}}{B} + (n^2 - 2n - 1)\frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{F}}{BF} + \frac{\ddot{F}}{F} = -\frac{1}{B^2}. \tag{42}$$

Adding these, we get

$$\frac{\ddot{B}}{B} + (n+1)\frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} = -\frac{1}{(n-1)B^2}. \tag{43}$$

Using power law relation between F and a, we obtain

$$F = kB^{n+2}. \tag{44}$$

Thus we get a differential equation with one unknown,

$$\frac{\ddot{B}}{B} + (2n+3)\frac{\dot{B}^2}{B^2} = -\frac{1}{(n-1)B^2}. \tag{45}$$

Put $\dot{B} = g(B)$ in this equation, we get

$$\frac{dg^2}{dB} + \frac{4n+6}{B}g^2 = -\frac{2}{(n-1)B}, \tag{46}$$

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which leads to the solution

$$g^2 = \frac{c_2}{B^{4n+6}} - \frac{1}{(n-1)(2n+3)},$$ (47)

where c_2 is an integration constant. Hence the solution becomes

$$ds^2 = \left(\frac{1}{\frac{c_2}{B^{4n+6}} - \frac{1}{(n-1)(2n+3)}}\right)dB^2 - B^{2n}dr^2 - B^2(d\theta^2 + \sinh^2\theta\phi^2),$$ (48)

which takes the form

$$ds^2 = \left(\frac{1}{\frac{c_2}{T^{4n+6}} - \frac{1}{(n-1)(2n+3)}}\right)dT^2 - T^{2n}dR^2 - T^2(d\Theta^2 + \sinh^2\theta\phi^2).$$ (49)

where $B = T$, $r = R$, $\theta = \Theta$ and $\phi = \Phi$.

The average Hubble parameter becomes here

$$H = \left(\frac{n+2}{3}\right)\left[\frac{c_2}{T^{4n+8}} - \frac{1}{(n-1)(2n+3)T^2}\right]^{\frac{1}{2}}.$$ (50)

while the volume scale factor turns out to be same as for the Bianchi type I spacetime. The expansion scalar θ is given by

$$\theta = (n+2)\left[\frac{c_2}{T^{4n+8}} - \frac{1}{(n-1)(2n+3)T^2}\right]^{\frac{1}{2}},$$ (51)

while the shear scalar σ becomes

$$\sigma^2 = \frac{1}{3}(n-1)^2\left[\frac{c_2}{T^{4n+8}} - \frac{1}{(n-1)(2n+3)T^2}\right].$$ (52)

Moreover, the function of Ricci scalar, $f(R)$, can be found by using Eq.(4)

$$f(R) = \frac{k}{2}\left[T^{n+2}R - \frac{3(2n^2+7n+6)}{2n^2+n-3}T^n\right].$$ (53)

It follows from Eq.(38) that

$$R = 2\left[\frac{c_2(n^2+6n+5)}{T^{4n+8}} + \frac{1}{T^2}\left(\frac{3n^2+2n-2}{2n^2+n-3}\right)\right],$$ (54)

which clearly indicates that $f(R)$ cannot be explicitly written in terms of R. However, for a special case when $n = -5$, $f(R)$ turns out to be

$$f(R) = \frac{k}{12\sqrt{3}}R^{5/2}$$

This gives $f(R)$ only as a function of R.

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5 Kantowski-Sachs Solution

The line element of Kantowski-Sachs spacetime is

$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 - \sin^2\theta d\phi^2], \tag{55}$$

where A and B are cosmic scale factors. The corresponding Ricci scalar is given by

$$R = -2[\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2}], \tag{56}$$

where dot denotes derivative with respect to t. The vacuum field equations for Kantowski-Sachs spacetime are given by

$$-\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} = 0, \tag{57}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0. \tag{58}$$

Here we also use Eq.(17) so that the field equations take the form

$$-\frac{2\ddot{B}}{B} + 2n\frac{\dot{B}^2}{B^2} + n\frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} = 0, \tag{59}$$

$$(n+1)\frac{\ddot{B}}{B} + (n^2 - 2n - 1)\frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{F}}{BF} + \frac{\ddot{F}}{F} = \frac{1}{B^2}. \tag{60}$$

Adding these, we get

$$\frac{\ddot{B}}{B} + (n+1)\frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{F}}{BF} = \frac{1}{(n-1)B^2}. \tag{61}$$

Using Eq.(23), we obtain one differential equation with one unknown,

$$\frac{\ddot{B}}{B} + (2n+3)\frac{\dot{B}^2}{B^2} = \frac{1}{(n-1)B^2}, \tag{62}$$

which gives

$$\dot{B}^2 = \frac{c_3}{B^{4n+6}} + \frac{1}{(n-1)(2n+3)}, \tag{63}$$

where c_3 is an integration constant. Hence the solution becomes

$$ds^2 = (\frac{1}{\frac{c_3}{T^{4n+6}} + \frac{1}{(n-1)(2n+3)}})dT^2 - T^{2n}dR^2 - T^2(d\Theta^2 + \sin^2\theta\phi^2). \tag{64}$$

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where $B = T$, $r = R$, $\theta = \Theta$ and $\phi = \Phi$.

The average Hubble parameter becomes here

$$H = (\frac{n+2}{3})[\frac{c_3}{T^{4n+8}} + \frac{1}{(n-1)(2n+3)T^2}]^{\frac{1}{2}}. \tag{65}$$

while the volume scale factor turns out to be same as for the Bianchi types I and III spacetimes. The expansion scalar θ is given by

$$\theta = (n+2)[\frac{c_3}{T^{4n+8}} + \frac{1}{(n-1)(2n+3)T^2}]^{\frac{1}{2}}, \tag{66}$$

while the shear scalar σ turns out to be

$$\sigma^2 = \frac{1}{3}(n-1)^2[\frac{c_3}{T^{4n+8}} + \frac{1}{(n-1)(2n+3)T^2}]. \tag{67}$$

The function of Ricci scalar becomes

$$f(R) = \frac{k}{2}[T^{n+2}R + \frac{3(2n^2+7n+6)}{2n^2+n-3}T^n], \tag{68}$$

while Ricci scalar takes the form

$$R = 2[\frac{c_3(n^2+6n+5)}{T^{4n+8}} - \frac{1}{T^2}(\frac{3n^2+2n-2}{2n^2+n-3})]. \tag{69}$$

For a special case when $n = -5$, $f(R)$ turns out to be

$$f(R) = \frac{k}{4\sqrt{3}}|-R|^{5/2}.$$

6   Concluding Remarks

The main purpose of this paper is to study some Bianchi type cosmological models in metric $f(R)$ gravity. We find exact solution of the vacuum field equations for Bianchi type I, III and Kantowski-Sachs spacetimes. Initially, the field equations look complicated but lead to a solution using some assumptions. The first assumption is that the expansion scalar θ is proportional to the shear scalar σ. It gives $A = B^n$, where A, B are the metric coefficients and n is an arbitrary constant. Secondly, the power law relation between F and a is used to find the solution. Some important cosmological physical

quantities for the solutions such as expansion scalar θ, shear scalar σ^2 and average Hubble parameter are evaluated. The analysis of these parameters in the light of solar system constraints is not done in this paper. However, it would be worthwhile to check the consistency of these parameters with Wilkinson Microwave Anisotropy Probe (WMAP) data. Also, the solutions can be compared with Lematre-Tolman cosmology [33] that describes the inhomogeneity in the universe on many scales.

The general function of Ricci scalar, $f(R)$, is also constructed in each case. For some special cases, it is found that $f(R)$ models include some root powers of the Ricci scalar. It has been shown [40] that the model $f(R) = R^{3/2}$ is consistent with cosmological results. In particular, it is possible to obtain flat rotation curves for galaxies and consistency with solar system tests. However, more work has still to be done to find the viability of these models and consistency with solar system.

The models of the universe in Eqs.(29,49,64) are non-singular at $T = 0$. The physical parameters H, θ and σ are all infinite at this point but the volume scale factor V vanishes. The general function of the Ricci scalar is finite while the spatial part of the metric vanish at $T = 0$. The expansion stops for $n = -2$ in all models. The models indicate that after a large time the expansion will stop completely and the universe will achieve isotropy. The isotropy condition, i.e., $\frac{\sigma^2}{\theta} \to 0$ as $T \to \infty$, is also satisfied in each case. Thus we can conclude from these observations that the models start their expansion from zero volume at $T = 0$ and the volume increases with the passage of time.

References

[1] Riess, A.G. et al. (Supernova Search Team): Astron. J. **116**(1998)1009; Bennett., C.L. et al.: Astrophys. J. Suppl. **148**(2003)1; Riess, A.G. et al.: Astrophys. J. **607**(2004)665.

[2] Spergel, D.N. et al.: Astrophys. J. Suppl. **148**(2003)175; ibid. **170**(2007)377.

[3] Tegmartk, M. et al.: Phys. Rev. **D69**(2004)103501.

[4] Chiba, T., Smith, T.L. and Erickcek, A.L.: Phys. Rev. **D75**(2007)124014.

[5] Capozziello, S., Martin-Moruno, P. and Rubano, C.: Phys. Lett. **B664**(2008)12.

[6] Nojiri, S. and Odintsov, S.D.: *Problems of Modern Theoretical Physics*, A Volume in honour of Prof. Buchbinder, I.L. in the occasion of his 60th birthday, p.266-285, (TSPU Publishing, Tomsk), arXiv:0807.0685.

[7] Nojiri, S. and Odintsov, S.D.: Int. J. Geom. Meth. Mod. Phys. **115**(2007)4.

[8] Lobo, F.S.N. and Oliveira, M.A.: Phys. Rev. **D80**(2009)104012.

[9] Cognola, G., Elizalde, E., Nojiri, S., Odintsov, S.D. and Zerbini, S.: JCAP **0502**(2005)010.

[10] Multamäki, T. and Vilja, I.: Phys. Rev. **D74**(2006)064022.

[11] Multamäki, T. and Vilja, I.: Phys. Rev. **D76**(2007)064021.

[12] Sharif, M. and Kausar, H.R.: *Non-vacuum Static Spherically Symmetric Solutions in $f(R)$ Gravity*, submitted for publication.

[13] Capozziello, S., Stabile, A. and Troisi, A.: Class. Quantum Grav. **24**(2007)2153.

[14] Hollenstein, L. and Lobo, F.S.N.: Phys. Rev. **D78**(2008)124007.

[15] Azadi, A., Momeni, D. and Nouri-Zonoz, M.: Phys. Lett. **B670**(2008)210.

[16] Momeni, D. and Gholizade, H.: Int. J. Mod. Phys. **D18**(2009)1.

[17] Reboucas, M.J. and Santos, J.: Phys. Rev. **D80**(2009)063009.

[18] Sharif, M. and Shamir, M.F.: Mod. Phys. Lett. **A25**(2010)1281.

[19] Babichev, E. and Langlois, D.: Phys. Rev. **D80**(2009)121501.

[20] Lorenz-Petzold, D.: Astrophys. Space Sci. **85**(1982)59.

[21] Hanquin, J.L. and Demaret, J.: Class. Quantum Grav. **1**(1984)291.

[22] Tikekar, R. and Patel, L.K.: Gen. Relativ. Grav. **24**(1992)397.

[23] Yavuz, I. and Yilmaz, I.: Astrophys. Space Sci. **245**(1996)131.

[24] Chakraborty, S., Chakraborty, N.C. and Debnath, U.: Int. J. Mod. Phys. **D12**(2003)325.

[25] Reddy, D.R.K., Adhav, K.S., Katore, S.D. and Wankhade, K.S.: Int. J. Theor. Phys. **48**(2009)2884.

[26] Christodoulakis, T. and Terzis, P.A.: Class. Quantum Grav. **24**(2007)875.

[27] Bagora, A.: Astrophys. Space Sci. **319**(2009)155.

[28] Kumar, S. and Singh, C.P.: Astrophys. Space Sci. **312**(2007)57.

[29] Moussiaux, A., Tombal, P. and Demaret, J.: J. Phys. A: Math. Gen. **14**(1981)277.

[30] Xing-Xiang, W.: Chin. Phys. Lett. **22**(2005)29.

[31] Wang, X.: Astrophys. Space Sci. **298**(2005)433.

[32] Upadhaya, R.D. and Dave, S.: Braz. J. Phys. **38**(2008)4.

[33] Hellaby, C.: Proceedings of Science, 5th International School on Field Theory and Gravitation, April 20-24, 2009, arXiv:0910.0350.

[34] Kumar, S. and Singh, C.P.: Int. J. Theor. Phys. **47**(2008)1722.

[35] Singh, T. and Agrawal, A.K.: Astrophys. Space Sci. **182**(1991)289.

[36] Adhav, K.S., Ugale, M.R., Kale, C.B. and Bhende, M.P.: Int. J. Theor. Phys. **48**(2009)178.

[37] Paul, B.C., Debnath, P.S. and Ghose, S.: Phys. Rev. **D79**(2009)083534.

[38] Sharif, M. and Shamir, M.F.: Class. Quantum Grav. **26**(2009)235020.

[39] Sharif, M. and Shamir, M.F.: Gen. Relativ. Gravit. (to appear), doi:10.1007/s10714-010-1005-5.

[40] Capozziello, S., Piedipalumbo, E., Rubano, C. and Scudellaro, P.: Astron. Astrophys. **505**(2009)21.